Seminonlinear matrix boundary-value problem

Sergey Chuiko, *Slavyansk, Ukraine*
Denis Sysoev, *Slavyansk, Ukraine*

We establish necessary and sufficient conditions for the existence of solutions

$$Z(t,\varepsilon):\ Z(\cdot,\varepsilon)\in\mathbb{C}^1[a;b],\quad Z(t,\cdot)\in\mathbb{C}[0;\varepsilon_0],\quad Z(t,\varepsilon)\in\mathbb{R}^{\alpha\times\beta}$$

of a nonlinear matrix differential equation [1,2]

$$Z'(t,\varepsilon)=AZ(t,\varepsilon)+Z(t,\varepsilon)B+F(t,\varepsilon)+\varepsilon\,\Phi(Z(t,\varepsilon),\mu(\varepsilon),t,\varepsilon) \qquad (1)$$

with a boundary condition

$$\mathcal{L}Z(\cdot,\varepsilon)=\mathcal{A}+\varepsilon J(Z(\cdot,\varepsilon),\mu(\varepsilon),\varepsilon),\quad \mathcal{A}\in\mathbb{R}^{\delta\times\gamma},\quad \alpha\neq\beta\neq\delta\neq\gamma. \qquad (2)$$

We seek the solution of the matrix boundary-value problem (1), (2) in a small neighborhood of the generating problem

$$Z_0'(t,\varepsilon)=AZ_0(t,\varepsilon)+Z_0(t,\varepsilon)B+F(t,\varepsilon),\quad \mathcal{L}Z_0(\cdot,\varepsilon)=\mathcal{A}. \qquad (3)$$

Here, $A\in\mathbb{R}^{\alpha\times\alpha}$ and $B\in\mathbb{R}^{\beta\times\beta}$ are constant matrices. Assume that the nonlinear matrix operator $\Phi(Z(t,\varepsilon),\mu(\varepsilon),t,\varepsilon):\ \mathbb{R}^{\alpha\times\beta}\to\mathbb{R}^{\alpha\times\beta}$ is Frechet differentiable with respect to the first argument in a small neighborhood of the solution of the generating problem and continuously differentiable with respect to μ in a small neighborhood of the solution of the generating problem (3) and the initial value $\mu_0(\varepsilon)$ of the eigenfunction $\mu(\varepsilon)$. The nonlinearity $\Phi(Z(t,\varepsilon),\mu(\varepsilon),t,\varepsilon)$ and inhomogeneity of the generating problem $F(t,\varepsilon)$ are regarded as continuous in t on a segment $[a,b]$ and in the small parameter ε on a segment $[0,\varepsilon_0]$. In addition, $\mathcal{L}Z(\cdot,\varepsilon)$ is a linear bounded matrix functional: $\mathcal{L}Z(\cdot,\varepsilon):\ \mathbb{C}^1[a;b]\to\mathbb{R}^{\delta\times\gamma}$. The nonlinear matrix functional $J(Z(\cdot,\varepsilon),\mu(\varepsilon),\varepsilon):C[a,b]\to\mathbb{R}^m$ is continuously differentiable with respect to Z in a small neighborhood of the solution of the generating problem (3), continuously differentiable with respect to μ in a small neighborhood of the solution of the generating problem (3) and the initial value $\mu_0(\varepsilon)$ of the eigenfunction $\mu(\varepsilon)$; and continuous in the small parameter ε on the segment $[0,\varepsilon_0]$.

[1] Boichuk A. A., Samoilenko A. M. Generalized Inverse Operators and Fredholm Boundary-value Problems 2-nd edition, Walter de Gruyter GmbH & Co KG, 2016.

[2] Chuiko S. M., Sysoev D. V. Weakly nonlinear matrix boundary-value problem in the case of parametric resonance // Journ. of Math. Sciences. – 2017. – 223. – 3. – pp. 337–350.