Mathematical modeling of the bioactive arterial wall

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Due to their bioactivity, the vessel walls can respond to the elevation of the blood pressure and wall shear stress [1]. The mechanical model of the bioactive arterial wall is based on the rheological equation of the wall [1]

$$\Lambda_R \frac{\partial R}{\partial t} + R = \Lambda_P \frac{\partial P}{\partial t} + (F_1(P) - F_2(C))\,\Phi(b), \tag{1}$$

where C and b are concentrations of Ca^{++} and NO, R and P are the radius of the vessel and the blood pressure in it, Λ_R, Λ_P, $F_1(P)$, $F_1(P)$ and $\Phi(b)$ are known empirical functions.

The influence of the Ca^{++} on the smooth muscle cells is governed by the kinetic equation [2]

$$\alpha \frac{\partial C}{\partial t} = -C + \psi(\sigma) + \beta \frac{\partial P}{\partial t}, \tag{2}$$

where $\sigma = PR/h$ is the circumferential stress, h is the wall thickness, α and β are constants.

Distribution of the NO is giverned by the diffusion equation

$$\frac{\partial b}{\partial t} = D_b \nabla^2 b - k b^{\xi}, \tag{3}$$

where D_b is the diffusion coefficient, k and ξ are constants.

The momentum equation for the wall has been taken in the form [2]

$$2\pi R \frac{\partial R}{\partial t} = \frac{\pi}{8\mu} \frac{\partial}{\partial t} \left(R^4 \frac{\partial P}{\partial x} \right), \tag{4}$$

where μ is the blood viscosity.

The system of partial differential equations (1)-(4) has been studied numerically by the finire difference method and iterations over time at a wide set of material parameters. Different regimes of the flow control by the bioactive wall are discussed.

[1] Zamir M. The Physics of Pulsatile Flow. Springer, – 2000.

[2] Regirer S. A., Shadrina N. X. Mathematical models of nitric oxide transport in a blood vessel. // Biofizika. – 2005. - Vol.50(3). – pp. 515-536.