Weakly and strongly nilpotent control systems. Examples among the Goursat flags

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Control systems linear in controls, with linearly independent vector field' generators, sometimes happen to be locally nilpotentizable. That is, to locally possess bases that generate (over reals, not over functions) nilpotent algebras of vector fields. The existence of a nilpotent basis may be somehow mischievously hidden in the nature of a system. When it exists and is at hand, a number of key control problems related with the system (e. g., motion planning) become much simpler. We call such systems *weakly* nilpotent. When a system Σ is given globally on a manifold M, we call weakly nilpotent those points in M, around which Σ is weakly nilpotent.

In turn, *strongly* nilpotent are those points p in M, around which Σ is equivalent to its *nilpotent approximation* at p. Naturally, 'strongly' implies 'weakly', but not vice versa: 'strongly' appears to be a much more stringent property.

An important class of weakly nilpotentizable distributions are *Goursat* distributions – members of Goursat *flags* which live on so-called Monster Manifolds ([1]). Local nilpotent bases found for Goursat distributions permit much more – to compute the *nilpotency orders* (sometimes also called 'indices', sometimes 'steps') of the generated real Lie algebras, see [2]. Those Lie algebras are sometimes called 'Kumpera-Ruiz' after the names of the discoverers of Goursat's singularities as such. A big problem, with only partial answers known to-date, reads

What points in the Goursat Monster Tower are strongly nilpotent ?

What are the dimensions of the Kumpera-Ruiz algebras, hidden in the Goursat distributions ?

(The nilpotency orders of the Kumpera-Ruiz algebras are tractable, but not their real dimensions.)

[1] Richard Montgomery & Michail Zhitomirskii. Points and Curves in the Monster Tower. *Memoirs of the American Mathematical Society* **956** (2010).

[2] Piotr Mormul. Goursat distributions not strongly nilpotent in dimensions not exceeding seven. Volume **281** (2003) of *Lecture Notes in Control and Information Sciences*, 249 – 261.