On the integration of nonlinear differential equation

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The Lax system of this form is studied in this paper

$$\begin{cases} [a(x),\gamma(x)]=0, \quad x\in[0,l], \\ \gamma'(x)=i[a(x),\sigma_2], \quad x\in[0,l], \\ \gamma(0)=\gamma^+, \end{cases} \tag{1}$$

where $a(x)$ – spectral matrix measure, $\gamma(x),\sigma_2,\gamma^+$ – self-conjugate $n\times n$ matrices, and

$$a(x)\geqslant 0, \qquad tra(x)\equiv 1, \qquad x\in[0,l].$$

The solution of this system $\gamma(x)$ is used in construction of triangular models of commutative systems of operators [1].

Proposition 1. *Let $\sigma_2=diag(b_1,...,b_n)$, $\quad\gamma^+=\alpha_1\sigma_2+\alpha_0 I+iC$, where $\alpha_1,\alpha_0\in\mathbb{R}$, matrix $C=(c_{jk})_{j,k=1}^n=-C^*$ and $c_{jj}=0$, $j\in\{1,\ldots,n\}$.*

Let further $\kappa_0,\ \kappa_1,\ \kappa_2\in L^1[0,l]$ – are real-valued functions. Then pair $\{a(\cdot),\gamma(\cdot)\}$, where $a(x)=\kappa_2(x)\gamma(x)^2+\kappa_1(x)\gamma(x)+\kappa_0(x)$, $x\in[0,l]$, and $\gamma(\cdot)=(\gamma_{jk}(\cdot))_{j,k=1}^n$, is the solution of the (1) if and only if $x\in[0,l]$ the following equations are completed

$$\begin{aligned} \gamma_{jj}(x) &= \gamma_{jj}^+, \quad j\in\{1,\ldots,n\}, \\ \gamma_{jk}(x) &= ie^{i(b_j-b_k)\left(K_1(x)+(\gamma_{jj}^++\gamma_{kk}^+)K_2(x)\right)}y_{jk}(x), \quad j\neq k, \end{aligned}$$

where

$$K_j(x):=\int_0^x \kappa_j(t)dt, \quad j\in\{1,2\},$$

and the functions $y_{jk}(\cdot)$, $j\neq k$, satisfy the system

$$\begin{cases} y'_{jk}(x) &= (b_k-b_j)\kappa_2(x)\sum\limits_{s=1,s\neq j,k}^n y_{js}(x)y_{sk}(x), \quad x\in[0,l], \quad j\neq k, \\ y_{kj}(x) &= \overline{-y_{jk}(x)}, \quad x\in[0,l], \quad j\neq k, \\ y_{jk}(0) &= c_{jk}, \quad j\neq k. \end{cases} \tag{2}$$

At that, if $c_{jk}\in\mathbb{R}$, $j\neq k$, then any solution of the system (2) is real-valued.

[1] Zolotarev V. A. Analytical methods of spectral represantations of non-selfadjoint and non-unitary operators. – Kharkov: KhNU, 2003. – 342 pp. (Russian).

[2] Zolotarev V. A. Functional models of commutative systems of linear operators and de Branges spase on the Riemannian surface.//Matematicheskiy sbornik, – 2009. Vol. 200. – **3**. – Pp. 31-48. (Russian).