Application of the method of lines to discretize problems of controllability for the partial differential equations, representing processes in power installations

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Suitable controlling programs, providing required state parameters during an operation in different modes, must be constructed for power installations, and it is an interesting not fully explored area for using the controllability theory.

Mathematical models of processes in power installations, including the heat conduction and others, can be represented using partial differential equations. In a point $\mathbf{x}$ we have a state vector $\mathbf{y}(\mathbf{x}, t)$, changing during a time $t \geq 0$ from an initial state $\mathbf{y}_0$ in corresponding with the controlling program, represented by a vector $\mathbf{u}(t)$, properties of the process in a domain Ω and an environment influence at a boundary Γ, represented by operators $\mathbf{A}(\mathbf{y}, \mathbf{u})$ and $\mathbf{B}(\mathbf{y}, \mathbf{u})$:

$$\partial\mathbf{y}/\partial t = \mathbf{A}(\mathbf{y}, \partial\mathbf{y}/\partial\mathbf{x}, \mathbf{u}),\ \mathbf{y}(x, 0) = \mathbf{y}_0, \mathbf{x} \in \Omega, \quad \mathbf{B}(\mathbf{y}, \mathbf{u}) = 0, \mathbf{x} \in \Gamma. \quad (1)$$

Mathematical model (1) is necessary to build the controlling program $\mathbf{u}(t)$, allowing to change the initial state of the process to a given state $\mathbf{y}_T$ during a minimal time T under required limiting conditions, represented in an operator $\mathbf{C}(\mathbf{y}, \mathbf{u})$:

$$\mathbf{u}(t): \quad \mathbf{y}(\mathbf{x}, T) = \mathbf{y}_T,\ \mathbf{C}(\mathbf{y}, \mathbf{u}) \geq 0,\ T \to \min. \quad (2)$$

To reduce the problem (1), (2), we use the spatial grid with nodes $\mathbf{x}_k \in \Omega$ and nodal values $\mathbf{y}_k(t) = \mathbf{y}(\mathbf{x}_k, t)$, $k = 1, 2, ..., n$. Following the method of lines [1], we use a finite differences technique in nodes $\mathbf{x}_k \in \Omega$ only for the differential operator $\partial\mathbf{y}/\partial\mathbf{x}$ and we reduce the problem (1), (2) to a view:

$$d\bar{\mathbf{y}}/dt = \bar{\mathbf{A}}(\bar{\mathbf{y}}, \mathbf{u}),\ \bar{\mathbf{y}}(0) = \bar{\mathbf{y}}_0,\ \mathbf{u}(t) : \bar{\mathbf{y}}(T) = \bar{\mathbf{y}}_T,\ \bar{\mathbf{C}}(\bar{\mathbf{y}}, \mathbf{u}) \geq 0,\ T \to \min, \quad (3)$$

where $\bar{\mathbf{y}}$ is a vector, including the nodal values $\mathbf{y}_1,\ \mathbf{y}_2,\ ...,\ \mathbf{y}_n$

Thus, the method of lines give us the opportunities to discretize the problem of controllability for partial differential equations, representing processes in power installations, and allows to reduce it to the controllability of the ordinary differential equations considered, for example, in [2].

[1] Fletcher C.A.J. Computational techniques for fluid dynamics. 1 Fundamental and general techniques, Springer-Verlag, 1988,1991: 1-401.

[2] Korobov V. I. The method of controllability function (Russian), R&C Dynamics, M.-Izhevsk, 2007: 1-576.